Lesson Opener

Find each product.

1. $\begin{bmatrix} -1 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1+0 & 0+5 \\ 3+0 & 0+8 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 3 & 8 \end{bmatrix}$

   $A \quad I = A$

2. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 6 \\ 7 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 4+0 & 0+0 & 6+0 \\ 0+7 & 0-5 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 6 \\ 7 & -5 & 4 \end{bmatrix}$

   $I \quad A = A$

   $\begin{bmatrix} 4 & 0 & 6 \\ 7 & -5 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2×3

3. $\begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & -1 \\ -2 & 3 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+0+0 & 0-3+0 & 0+0+1 \\ 4+0+0 & 0+2+0 & 0+0-1 \\ -2+0+0 & 0+3+0 & 0+0-3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & -1 \\ -2 & 3 & -3 \end{bmatrix}$

   $A \quad I = A$

What did you notice about each product?

It is the same as the given matrix, A.

An identity matrix of multiplication is a <u>square</u> matrix with <u>ones (1)</u> on the main diagonal (upper left to lower right) and <u>zeros (0)</u> for all other entries.

Note: If A is a square matrix with dimensions $n \times n$ and I is the identity matrix with dimensions $n \times n$, then $AI = IA$.

$[1]$ 1×1

2 X 2 multiplicative identity: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3 X 3 multiplicative identity: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Write the 4 X 4 identity matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Find each product.

4. $\begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -7+8 & -14+14 \\ 4-4 & 8-7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{3} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3-2 & -1+1 \\ 6-6 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

What do you notice?

The result is the identity.

If A and A^{-1} are inverse matrices, then <u>$A \cdot A^{-1} = I$</u> and <u>$A^{-1} \cdot A = I$</u>.

A^{-1} is the inverse of matrix A.

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Are the following matrices inverses? Show work and explain.

6. $\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4-3 & 3-3 \\ -4+4 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since the product of the matrices is an identity, the matrices are inverses.

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The inverse matrix is $A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $|A|=ad-bc$ and $|A|\neq 0$.

Find the inverse matrix if it exists. If it does not exist, explain why.

7. $A=\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$

$|A| = 12-10 = 2$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ 1 & 2 \end{bmatrix}$

8. $B=\begin{bmatrix} 8 & -4 \\ 5 & -3 \end{bmatrix}$

9. $C=\begin{bmatrix} -2 & 3 \\ -8 & 12 \end{bmatrix}$

Solving Matrix Equations

Recall: Solve the linear equation $\frac{1}{2}x+4=9$

1. Find unknown matrix X such that

$$\underset{\boldsymbol{A}}{\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}}\underset{\boldsymbol{X}}{X} = \underset{\boldsymbol{B}}{\begin{bmatrix} 9 & 12 & 0 \\ -4 & 5 & -2 \end{bmatrix}}$$

So what have we found? How can we check the answer?

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2. Solve the matrix equation $\begin{bmatrix} -4 & 2 \\ 8 & 1 \end{bmatrix} X = \begin{bmatrix} -16 & 6 \\ 22 & 13 \end{bmatrix}$

3. Solve the matrix equation $\begin{bmatrix} 4 & -3 \\ 6 & -2 \end{bmatrix} X - \begin{bmatrix} -1 & 1 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 2 \end{bmatrix}$

A system of linear equations can be rewritten as a matrix equation.

$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$ can be written as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$

coefficient matrix → $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; **variable matrix** → $\begin{bmatrix} x \\ y \end{bmatrix}$; **constant matrix** → $\begin{bmatrix} e \\ f \end{bmatrix}$

Rewrite the system of equations as a matrix equation and use inverse matrices to solve for x and y.

4. $\begin{cases} 2x + 3y = 15 \\ x - 2y = -17 \end{cases}$

5. $\begin{cases} 4x + 2y = -10 \\ x - 3y = 15 \end{cases}$

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Matrices and Graphing Calculator
Find the inverse of a 3 X 3 Matrix using technology.

6. $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

7. $B = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 2 \\ 1 & 6 & 5 \end{bmatrix}$

Solve a 3 X 3 system using inverse matrices and technology; SOME work is required!

8. $$\begin{aligned} 2x + 3y + 4z &= 7 \\ -x + 5y + 2z &= 6 \\ -3x + 6y &= 3 \end{aligned}$$

Inverse and Identity Matrices **Solving Equations using Inverses**

You must show all work for 2x2 matrices and systems.

Inverse Matrices

For each matrix state if an inverse exists.

1) $\begin{bmatrix} -9 & -9 \\ -2 & -2 \end{bmatrix}$

2) $\begin{bmatrix} -2 & 1 \\ -6 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 4 & -5 \\ -9 & 6 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 0 \\ -6 & 4 \end{bmatrix}$

Find the inverse of each matrix.

5) $\begin{bmatrix} 11 & -5 \\ 2 & -1 \end{bmatrix}$

6) $\begin{bmatrix} 0 & -2 \\ -1 & -9 \end{bmatrix}$

7) $\begin{bmatrix} -1 & 7 \\ -1 & 7 \end{bmatrix}$

8) $\begin{bmatrix} 1 & -1 \\ -6 & -3 \end{bmatrix}$

9) $\begin{bmatrix} 1 & -8 \\ 1 & -5 \end{bmatrix}$

10) $\begin{bmatrix} -6 & 6 \\ 3 & -3 \end{bmatrix}$

11) $\begin{bmatrix} 6 & 1 & 0 \\ -2 & -2 & 2 \\ -5 & 0 & 0 \end{bmatrix}$

12) $\begin{bmatrix} -2 & -2 & -2 \\ 0 & 2 & 4 \\ 2 & 5 & 6 \end{bmatrix}$

Solve each system, if possible, using inverse matrices.

13) $2x - 4y = 26$
$3x + 6y = -21$

14) $4x + 3y = 12$
$2x + 4y = -4$

15) $-6x - 4y - 3z = 6$
$-3x + 2y - 6z = -9$
$2x + y = -1$

16) $3x + z = -5$
$6x - y + 2z = -9$
$-2x + 3y + 3z = 4$

17) $-x + 3y = -15$
$2x - 3y - z = 23$
$-3x - 3y + 5z = -22$

18) $2x + 4y - 3z = -17$
$3x - 4y + 2z = 8$
$-6x - 4y + z = 27$

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