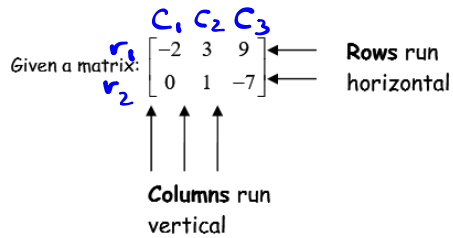


ACC Precalculus
Introduction to Matrices

Name _____
Date _____ Block _____

A matrix is a rectangular arrangement of numbers in rows and columns.
The dimensions of a matrix with r rows and c columns are $r \times c$. The numbers inside the matrix are the entries or elements.



To state the size/order/dimensions of the matrix:
row x column
The matrix to the left is a 2 x 3 matrix

State the size/order/dimensions of the following matrices:

1. $\begin{bmatrix} 6 & -9 & 11 & 4 \\ 3 & 7 & -6 & 0 \end{bmatrix}$ 2. $\begin{bmatrix} 9 & 6 \\ 3 & -8 \end{bmatrix}$ 3. $\begin{bmatrix} 6 & 1 \\ 0 & 5 \\ 9 & 5 \end{bmatrix}$ 4. $[-2 \ 4 \ 7]$
- 2x4 2x2 3x2 1x3

Some matrices have special names because of their dimensions or entries.

Name	Description	Example
Row Matrix	A matrix with only 1 row	$[3 \ -2 \ 0]$
Column Matrix	A matrix with only 1 column	$\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$
Square Matrix	A matrix with the same number of rows and columns	$\begin{bmatrix} 4 & -1 & 5 \\ 2 & 0 & 1 \\ 1 & -3 & 6 \end{bmatrix}$
Zero Matrix	A matrix whose entries are all zero	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If two matrices have the same dimensions and the same entries (or elements) in corresponding positions, then those matrices are said to be equal.

One example of equal matrices: $\begin{bmatrix} 18 & 2(3) \\ 2 & \\ 3 & -\frac{24}{3} \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 3 & -8 \end{bmatrix}$

To solve for a variable using equal matrices:

- Simplify one or both sides of the equation using matrix operations as needed.
- Set corresponding entries equal to each other and solve for the variable.

#5-6: Solve for x and y.

$$5. \begin{bmatrix} 6 & 5 \\ x+8 & 4 \\ 0 & 2y-1 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 14-x & 4 \\ 0 & -13-y \end{bmatrix}$$

$$x+8=14-x \quad 2y-1=-13-y$$

$$2x=6 \quad 3y=-12$$

$$\boxed{x=3} \quad \boxed{y=-4}$$

$$6. \begin{bmatrix} 12 & 3 \\ 6y & 5 \end{bmatrix} = \begin{bmatrix} -4x & 3 \\ 8x & 5 \end{bmatrix}$$

$$\frac{12}{-4} = \frac{-4x}{-4}$$

$$\boxed{x=-3}$$

$$6y=8x$$

$$6y=8(-3)$$

$$6y=-24$$

$$\boxed{y=-4}$$

Matrices Operations

Two or more matrices can be added or subtracted only if they have the same dimensions. To add or subtract matrices, simply add or subtract the corresponding entries.

Find the sum if possible. If not possible, state why.

$$7. \begin{bmatrix} 2 & -6 \\ 9 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -9 & 8 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 0 & 10 \end{bmatrix}$$

$$8. \begin{bmatrix} -3 & 6 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 8 \\ 7 \end{bmatrix} =$$

Not defined since the matrices have different dimensions.

In matrix Algebra, a real number is called a scalar. To multiply a matrix by a scalar, multiply each entry in the matrix by the scalar. This process is called scalar multiplication.

Find the product of $\frac{1}{2} \begin{bmatrix} 4 & -3 \\ 7 & 2 \end{bmatrix}$.

$$= \begin{bmatrix} 2 & -\frac{3}{2} \\ \frac{7}{2} & 1 \end{bmatrix}$$

scalar →

Note: Matrix operations follow the order of operations rules. Perform the indicated operations.

$$9. 2 \begin{bmatrix} \frac{1}{2} & 1 \\ 2 & -3 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -6 & 8 \end{bmatrix} + \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -1 & 7 \end{bmatrix}$$

$$10. \frac{1}{2} \left(\begin{bmatrix} 4 & 5 & 1 \\ 7 & 4 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 3 & -5 \\ 5 & -1 & -12 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 6 & 2 & 6 \\ 2 & 5 & 14 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 3 \\ 1 & \frac{5}{2} & 7 \end{bmatrix}$$

Given the following matrices, answer the questions that follow:

$$A = \begin{bmatrix} 4 & 2 \\ 9 & 0 \\ -1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 4 \\ 3 & -1 & 7 \end{bmatrix} \quad C = \begin{bmatrix} 6 & -9 & 0 \\ 5 & 3 & -1 \\ 11 & 4 & 2 \end{bmatrix} \quad D = \begin{bmatrix} -4 & 3 & 7 \\ 1 & 2 & 0 \end{bmatrix} \quad E = \begin{bmatrix} -1 & 3 \\ 6 & 9 \\ 10 & 2 \end{bmatrix}$$

11. A+E $\begin{bmatrix} 3 & 5 \\ 15 & 9 \\ 9 & ? \end{bmatrix}$

12. B-D $\begin{bmatrix} 4 & -2 & -3 \\ 2 & -3 & 7 \end{bmatrix}$

13. A+C
 Not possible due to different dimensions

14. Solve for x and y.

$$2 \left(\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix} \right) = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

$$2 \begin{bmatrix} 3x+4 & 0 \\ 6 & 5-y \end{bmatrix} \rightarrow \begin{bmatrix} 6x+8 & 0 \\ 12 & 10-2y \end{bmatrix} = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

$$6x+8=26 \quad 10-2y=8$$

$$6x=18 \quad -2y=-2$$

$$\boxed{x=3} \quad \boxed{y=1}$$

Applications using Matrices

Use matrices to organize the following information about health care plans.

This Year For individuals, Comprehensive, HMO Standard, and HMO Plus cost \$694.32, \$451.80, and \$489.48, respectively. For families, the Comprehensive, HMO Standard, and HMO Plus plans cost \$1725.36, \$1187.76, and \$1248.12.

Next Year For individuals, Comprehensive, HMO Standard, and HMO Plus cost \$683.91, \$463.10, and \$499.27, respectively. For families, the Comprehensive, HMO Standard, and HMO Plus plans cost \$1699.48, \$1217.45, and \$1273.08.

	Comp.	Stand.	Plus	
Ind	694.32	451.80	489.48	Ind $\begin{bmatrix} 683.91 & 463.10 & 499.27 \end{bmatrix}$
Fam	1725.36	1187.76	1248.12	Fam $\begin{bmatrix} 1699.48 & 1217.45 & 1273.08 \end{bmatrix}$
<u>This Year</u>	A			<u>Next Year</u> B

A company offers the health care plans in the last example to its employees. The employees receive monthly paychecks from which health care payments are deducted. Use the matrices in the last example to write a matrix that shows the monthly changes in health care payments from this year to next year.

B-A : changes from this year to next year.

	Comp	STAND	PLUS
Ind	-10.41	11.30	9.79
Fam	-25.88	29.69	24.96

Please turn in before the end of class. *A matrix is a rectangular arrangement of numbers (information).*

1. What is a matrix? Describe and give an example of a row matrix, a column matrix, and a square matrix. Row: $[3 \ -47 \ 8 \ \frac{1}{2}]$ column: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Square: $[36]$

Your answers may vary!

2. Are the two matrices equal? Explain. $A = \begin{bmatrix} -6 & \frac{1}{2} \\ 4 & -5 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -6 & 0.5 \\ 4 & -5 \\ 3 & 5 \end{bmatrix}$
Yes, since they have the same dimensions and corresponding entries are equal.

3. To add or subtract two matrices, what must be true?

The matrices must have the same dimensions.

4. Use the matrices in question 2 to find $-2(A+B)$. $-2 \begin{bmatrix} -12 & 1 \\ 8 & -10 \\ 6 & 10 \end{bmatrix} = \begin{bmatrix} 24 & -2 \\ -16 & 20 \\ -12 & -20 \end{bmatrix}$

5. $\begin{bmatrix} 20 \\ -22 \\ 9 \end{bmatrix} - \begin{bmatrix} -11 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} 31 \\ -12 \\ 15 \end{bmatrix}$

6. $\begin{bmatrix} -6 & -7 & 4 \\ -4 & 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -5 & 8 \\ 9 & 12 & -9 \end{bmatrix} = \begin{bmatrix} -7 & -12 & 12 \\ 5 & 12 & -10 \end{bmatrix}$

7. $-4 \begin{bmatrix} 2 & 0 \\ -4 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 16 & 20 \end{bmatrix}$

8. $6 \begin{bmatrix} -5 & -1 \\ 2 & 0 \end{bmatrix} - 5 \begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} -30 & -6 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ -20 & 15 \end{bmatrix} = \begin{bmatrix} -25 & -6 \\ -8 & 15 \end{bmatrix}$

Solve for x and y.

9. $\begin{bmatrix} 10 & -3y \\ 6 & 13 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 6x & 13 \end{bmatrix}$ $6 = 6x$ $-3y = -15$
 $x = 1$ $y = 5$

10. $\begin{bmatrix} 12 & 3 \\ 6y & 5 \end{bmatrix} = \begin{bmatrix} -4x & 3 \\ 24 & 5 \end{bmatrix}$ $3 = -4x$ $6y = 24$
 $x = -\frac{3}{4}$ $y = 4$

11. $-2 \left(\begin{bmatrix} -3x & -1 \\ 4 & y \end{bmatrix} + \begin{bmatrix} 9 & -4 \\ -6 & 3 \end{bmatrix} \right) = \begin{bmatrix} 6 & 10 \\ 4 & -20 \end{bmatrix}$
 $-2 \begin{bmatrix} -3x+9 & -5 \\ -2 & y+3 \end{bmatrix} = \begin{bmatrix} 6x-18 & 10 \\ 4 & -2y-6 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 4 & -20 \end{bmatrix}$
 $6x-18=6$ $-2y-6=-20$
 $x=4$ $y=7$

ACC Precalculus
Multiplying Matrices

$r \times c$

Name _____

In order to multiply matrices $A \times B$, the number of columns in A must equal the number of rows in B. If the product is defined, the solution matrix will have the same number of rows as A, and the same number of columns as B.

State whether the product is defined. If it is defined, give the dimension of the product. If not defined, say why.

1. $A: 3 \times 2; B: 4 \times 3$

$r \times c; r \times c$
AB? No, since columns of A \neq rows of B.

-BA?

$4 \times 3; 3 \times 2 \rightarrow 4 \times 2$

2. $A: 3 \times 3; B: 3 \times 2$

AB? $3 \times 3; 3 \times 2 \rightarrow 3 \times 2$

BA?

$3 \times 2; 3 \times 3$
 $r \times c; r \times c$
No since columns of B \neq rows of A.

In order to find the product of two matrices, $A \times B$, multiply the rows in A times the columns in B.

3. $A = \begin{bmatrix} 6 & 2 \\ -1 & 3 \\ 0 & -4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 \\ 5 & -2 \end{bmatrix}$, find AB.

$3 \times 2; 2 \times 2 \rightarrow 3 \times 2$

$$\begin{bmatrix} \frac{-6+10}{r_1c_1} & \frac{12-4}{r_1c_2} \\ \frac{1+15}{r_2c_1} & \frac{-2-6}{r_2c_2} \\ \frac{0-20}{r_3c_1} & \frac{0+8}{r_3c_2} \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 16 & -8 \\ -20 & 8 \end{bmatrix}$$

$r_1c_1 = 6(-1) + 2(5)$
 $r_1c_2 = 6(2) + 2(-2)$
 $r_2c_1 = (-1)(-1) + (3)(5)$
etc.

4. $A = \begin{bmatrix} 7 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, find AB and BA.

$AB = \begin{bmatrix} 7 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 28-3 \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$

$1 \times 2; 2 \times 1$

$BA = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \begin{bmatrix} 7 & 3 \end{bmatrix} = \begin{bmatrix} 28 & 12 \\ -7 & -3 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 7 \\ -4 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -5 & -2 \\ 1 & 0 & 6 \end{bmatrix}$$

5. Find $(A+B)C$. Also find $AC+BC$.

$$\begin{bmatrix} 5 & 4 \\ -5 & 0 \end{bmatrix} \begin{bmatrix} 3 & -5 & -2 \\ 1 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 15+4 & -25+0 & -10+24 \\ -15+0 & 25+0 & 10+0 \end{bmatrix} = \begin{bmatrix} 19 & -25 & 14 \\ -15 & 25 & 10 \end{bmatrix}$$

$A+B$ C
 $2 \times 2; 2 \times 3 \rightarrow$ 2×3

$$AC = \begin{bmatrix} 12-3 & -20+0 & -8-18 \\ -3+2 & 5+0 & 2+12 \end{bmatrix} = \begin{bmatrix} 9 & -20 & -26 \\ -1 & 5 & 14 \end{bmatrix} \quad BC = \begin{bmatrix} 3+7 & -5+0 & -2+42 \\ -12-2 & 20+0 & 8-12 \end{bmatrix} = \begin{bmatrix} 10 & -5 & 40 \\ -14 & 20 & -4 \end{bmatrix}$$

$$AB+BC = \begin{bmatrix} 19 & -25 & 14 \\ -15 & 25 & 10 \end{bmatrix} \quad \text{So } (A+B)C = AC+BC$$

6. Find AB and BA .

$$AB = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 16 & 34 \\ -9 & -11 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 7 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 11 \\ -14 & 8 \end{bmatrix}$$

$$AB \neq BA$$

Let A , B , and C be matrices and k be a scalar.

Properties of Matrix Addition

Associative Property of Matrix Addition

$$(A+B)+C = \underline{A+(B+C)}$$

Commutative Property of Addition

$$A+B = \underline{B+A}$$

Distributive Property of Addition

$$k(A+B) = \underline{kA+kB}$$

Distributive Property of Subtraction

$$k(A-B) = \underline{kA-kB}$$

Properties of Matrix Multiplication

Associative Property of Matrix Multiplication

$$A(BC) = \underline{(AB)C}$$

Left Distributive Property

$$A(B+C) = \underline{AB+AC}$$

Right Distributive Property

$$(A+B)C = \underline{AC+BC}$$

Associative Property of Scalar Multiplication

$$k(AB) = \underline{(kA)B = A(kB)}$$

NOTE: There is NO Commutative Property of Multiplication for Matrices; generally $\underline{AB \neq BA}$

Multiplying Matrices Practice 1

Determine the dimensions of each matrix product.

1. $A_{2 \times 3} \cdot B_{3 \times 5}$

2×5

2. $G_{4 \times 7} \cdot C_{7 \times 1}$

4×1

3. $M_{2 \times 2} \cdot N_{3 \times 2}$

Product is undefined since columns of M does not equal rows of N.

Multiply the following matrices, if possible.

4. $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} 6 & -8 \\ 12 & -5 \end{bmatrix}$

$\begin{bmatrix} 48 & -31 \\ 108 & -67 \end{bmatrix}$

5. $\begin{bmatrix} 13 \\ 5 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 & 11 \\ 10 & 4 \end{bmatrix}$

$3 \times 1, 1 \times 3$

$\begin{bmatrix} 78 & 26 & 143 \\ 30 & 10 & 55 \\ 48 & 16 & 22 \end{bmatrix}$

6. $\begin{bmatrix} 9 & 8 \\ 4 & 3 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} -10 & -2 \\ -3 & -4 \end{bmatrix}$

$3 \times 2; 2 \times 2 \rightarrow 3 \times 2$

$\begin{bmatrix} -114 & -50 \\ -49 & -20 \\ -68 & -34 \end{bmatrix}$

7. $\begin{bmatrix} -5 & 12 \\ 8 & -3 \\ -9 & -6 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ -5 & 1 \\ 4 & 3 \end{bmatrix}$

$3 \times 2; 3 \times 2$

Undefined since columns of 1st matrix do not equal rows of 2nd matrix.

8. $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & -6 & 10 \\ 4 & -8 & 12 \end{bmatrix}$

$2 \times 2; 2 \times 3 \rightarrow 2 \times 3$

$\begin{bmatrix} 14 & -30 & 46 \\ 38 & -86 & 134 \end{bmatrix}$

Solve the following matrix multiplication word problems.

9. On two days, a store sold the following amounts of pencils, erasers, and binders.

	Pencils	Erasers	Binders	Price
Monday	48	7	9	$\begin{bmatrix} P \\ E \\ B \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.35 \\ 2.85 \end{bmatrix}$
Tuesday	54	10	6	

If the price for each pencil, eraser, and binder, respectively, is \$0.20, \$0.35, and \$2.85, how much was made each day?

multiply! $2 \times 3; 3 \times 1 \rightarrow 2 \times 1$

MON $\begin{bmatrix} \text{Sales} \\ 37.70 \end{bmatrix}$
 TUES $\begin{bmatrix} 31.40 \end{bmatrix}$

They made \$37.70 on Monday and \$31.40 on Tuesday.

10. Old MacDonald has three fruit farms. On these farms he grows peaches, apricots, plums, and apples. When picked, the fruit is sorted into layered boxes in which they will be sold. The chart below shows the number of boxes for each type of fruit.

Location	Peaches	Apricots	Plums	Apples
Farm 1	152	225	395	277
Farm 2	236	183	245	183
Farm 3	95	132	0	285

matrix A

matrix B

Suppose he sells peaches for \$27 a box, apricots for \$15 a box, plums for \$34 a box, and apples for \$17 a box. Find the income for each farm. How much will he make total?

Find AB

Income

Farm 1	25,618
2	20,558
3	9390

Old MacDonald will make a total of \$ 55,566.00.

11. In a three team track meet, the following numbers of 1st, 2nd, and 3rd place finishes were recorded.

School	First Place	Second Place	Third Place
Lee	4	10	6
Central	7	6	9
Clarke	8	3	4

If 5 points are awarded for 1st, 3 points for 2nd, and 1 point for 3rd, determine who won the track meet.

Points

1 st	5
2 nd	3
3 rd	1

Product

	1 st	2 nd	3 rd
Lee	4	10	6
central	7	6	9
clark	8	3	4

Points

1 st	5
2 nd	3
3 rd	1

Total Points

Lee	56
central	62
clark	53

Clark won the track meet with a total of 62 points.

4

Create Your Own Matrices I did not write actual answers but described what needs to be true about YOUR answers.

1. Create two matrices, A and B, such that $A + B$ is defined but AB is undefined. Find $A + B$; show why AB is undefined.

Matrices A and B must have the same dimensions, but they can't be square matrices.

2. Create two matrices, A and B, such that AB is defined but $A + B$ is undefined. Find AB ; explain why $A + B$ is not defined.

The matrices cannot have the same dimensions; the row of matrix A must be the same as the columns of matrix B.

3. Create two matrices, A and B, such that $A + B$ is defined AND AB is defined. Find $A + B$ and AB .

A and B must be square matrices with the same dimensions.

4. Create two matrices, A and B, such that AB is defined but BA is undefined. Find AB ; explain why BA is not defined.

The number of columns in A must be the same as the number of rows in B, BUT the number of rows in A cannot be the same as the number of columns in B.

For example, A is a 2×3 and B is a 3×1 .

5. Create two matrices, A and B, such that AB is defined AND BA is defined. Find AB and BA .

A and B need to be square matrices and have the same dimensions. For example, they are both 2×2 matrices, or both 3×3 , etc.