ACC Precalculus
Law of Cosines

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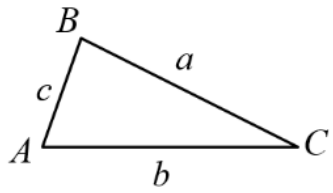
Date _____ Block _____

The Law of Cosines

The law of cosines is used to solve triangles given two sides and the included angle (SAS) or given three sides (SSS).
When given 3 sides of a triangle (SSS), you must find the angle opposite the largest side first!

In any triangle *ABC*,

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$



When the included angle is 90°, the law of cosines reduces to the Pythagorean theorem.

Example 1: In ΔABC, $a = 32$, $c = 48$, and $B = 125.2°$. Solve the triangle.

$b^2 = 32^2 + 48^2 - 2(32)(48)\cos(125.2°)$

$b^2 = 5098.8\ldots$

$b = \sqrt{ANS} \approx 71.4$

$\frac{71.4}{\sin 125.2°} = \frac{32}{\sin A}$

$\sin A = \frac{32\sin 125.2°}{71.4} \rightarrow A = \sin^{-1}(ANS) \approx 21.5°$

$C = 180 - (125.2 + 21.5) = 33.3$

$A = 21.5°$ $a = 32$
$B = 125.2°$ $b = 71.4$
$C = 33.3°$ $c = 48$

SAS
LOC
1 D

Example 2: Solve ΔRST, given $r = 3.5$, $s = 4.7$, and $t = 2.8$.

$s^2 = r^2 + t^2 - 2rt\cos S$

$4.7^2 = 3.5^2 + 2.8^2 - 2(3.5)(2.8)\cos S$

$(4.7^2 - 3.5^2 - 2.8^2) = -2(3.5)(2.8)\cos S$ (1st)

$\frac{(4.7^2 - 3.5^2 - 2.8^2)}{(-2(3.5)(2.8))} = \cos S$

$\cos S = \frac{(4.7^2 - 3.5^2 - 2.8^2)}{(-2\cdot 3.5\cdot 2.8)}$

$\cos S = -0.102$

$S = \cos^{-1}(ANS) = 95.9°$

$180 - (47.8 + 95.9) \rightarrow T = 36.3°$

$\frac{3.5}{\sin R} = \frac{4.7}{\sin 95.9}$

$\sin R = \frac{3.5\sin 95.9°}{4.7}$

$R = \sin^{-1}(ANS)$

$R = 47.8°$

$R = 47.8°$ $r = 3.5$
$S = 95.9°$ $s = 4.7$
$T = 36.3°$ $t = 2.8$

Use Heron's Formula to find the area of a triangle when given 3 sides (SSS).

Heron's Formula

The area K of a triangle with sides a, b, and c is

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

Area

where $s = \frac{1}{2}(a+b+c)$.

Note: s is the "semi-perimeter."

Example 3: Find the area of a triangle with side 5, 8, and 10.

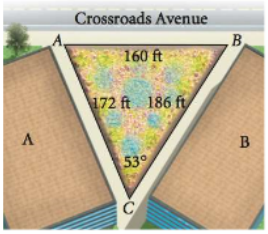
$s = \frac{1}{2}(5+8+10) = 11.5$

$K = \sqrt{11.5(11.5-5)(11.5-8)(11.5-10)}$

$= \sqrt{11.5(6.5)(3.5)(1.5)}$

$K \approx 19.8$ sq units

Example 4: A university landscaping architecture department is designing a garden for a triangular area in a dormitory complex. Two sides of the garden, formed by the sidewalks in front of buildings A and B, measure 172 ft and 186 ft, respectively, and together form a 53° angle. The third side of the garden, formed by the sidewalk along Crossroads Avenue, measures 160 ft. What is the area of the garden to the nearest square foot?



$s = \frac{1}{2}(172+160+186) = 259$

$K = \sqrt{259(87)(99)(73)}$

$K = 12,761$ sq ft

Related topic: Find the area of a triangle using sine. SAS

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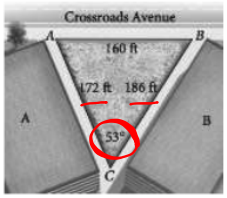
The area of any ΔABC is one half the product of the lengths of two sides and the sine of the included angle. Let K be the area of a triangle. Then

$K = \frac{1}{2}bc\sin A$ $K = \frac{1}{2}ab\sin C$ $K = \frac{1}{2}ac\sin B$

Choose the formula needed depending on the given information.

Example 7: A university landscaping architecture department is designing a garden for a triangular area in a dormitory complex. Two sides of the garden, formed by the sidewalks in front of buildings A and B, measure 172 ft and 186 ft, respectively, and together form a 53° angle. The third side of the garden, formed by the sidewalk along Crossroads Avenue, measures 160 ft. What is the area of the garden to the nearest square foot?



$K = \frac{1}{2}(172)(186)\sin 53°$

$K = 12{,}775$ sq ft

Compared to 12,761 sq ft

Unit 5 **Law of Sines and Law of Cosines** **Formulas**

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

$A = \sqrt{s(s-a)(s-b)(s-c)}$
$s = \frac{1}{2}(a+b+c)$

$K = \frac{1}{2}bc\sin A$ $K = \frac{1}{2}ac\sin B$ $K = \frac{1}{2}ab\sin C$

Complete p.10: #1-6, also p.12: 1, 3, 6, 7, 12, 13, and 15. You need to add this work to last night's work on separate paper. Answers below.

Answers to Law of Sines and Cosines Practice Worksheet

1) $m\angle C = 108.6°$, $m\angle A = 31.1°$, $m\angle B = 40.3°$
2) $m\angle F = 61°$, $m\angle D = 65°$, $e = 25$
3) $m\angle K = 47.3°$, $m\angle H = 49.1°$, $p = 16.1$
4) $m\angle T = 16°$, $m\angle R = 50°$, $s = 29.8$
5) $m\angle C = 59°$, $m\angle A = 30°$, $m\angle B = 91°$
6) $m\angle Y = 23.8°$, $m\angle Z = 115.6°$, $m\angle X = 41.4°$
7) $m\angle X = 51.9°$, $m\angle Y = 72.2°$, $z = 24.7$
8) $m\angle X = 30°$, $m\angle Y = 32°$, $m\angle Z = 118°$
9) $m\angle K = 19.3°$, $m\angle H = 50.7°$, $p = 34$
10) $m\angle F = 60.3°$, $m\angle D = 25.7°$, $e = 20.7$
11) $m\angle X = 137.1°$, $m\angle Y = 22.9°$, $x = 57.7$ cm
Or $m\angle X = 2.9°$, $m\angle Y = 157.1°$, $x = 4.3$ cm
12) $m\angle Q = 5.7°$, $r = 27.8$ m, $p = 24.8$ m
13) Not a triangle
14) $m\angle P = 60.6°$, $m\angle K = 30.4°$, $p = 31$ in
15) Not a triangle
16) $m\angle E = 116.6°$, $m\angle F = 32.4°$, $e = 41.7$ in
Or $m\angle E = 1.4°$, $m\angle F = 147.6°$, $e = 1.1$ in
17) $m\angle Y = 16°$, $m\angle Z = 14°$, $y = 16$ cm
18) $m\angle A = 49°$, $c = 11$ cm, $b = 32$ cm
19) $m\angle B = 88°$, $a = 12$ ft, $b = 39.4$ ft
20) $m\angle Q = 11.2°$, $q = 12$ in, $p = 22$ in

Answers to Area of Triangles

1) 29.4 units²
2) 18.5 units²
3) 17.2 units²
4) 7.4 units²
5) 22.8 units²
6) 54.3 units²
7) 22.4 units²
8) 46.5 units²
9) 34 units²
10) 61.7 units²
11) 85.7 units²
12) 44 units²
13) 7.7 units²
14) 73.8 units²
15) 23.4 units²
16) 116.5 units²
17) 17.2 units²
18) 35.2 units²
19) 71.6 units²
20) 113.6 units²