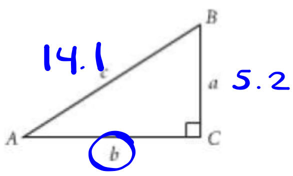
ACC Precalculus Name ______________________
Law of Sines Date ____________ Block ________

Lesson Opener/Recall

Solve the right triangle given $a = 5.2, c = 14.1$. Round answers to the nearest tenth.



$A = 21.6°$ $a = 5.2$

$B = 68.4°$ $b = 13.1$

$C = 90°$ $c = 14.1$

$\sin A = \frac{5.2}{14.1}$

$A = \sin^{-1}\left(\frac{5.2}{14.1}\right)$

$\sin B = \frac{b}{14.1}$

$5.2^2 + b^2 = 14.1^2$

$b^2 = 171.77 \to b = 13.1$

What are the 4 methods for proving that 2 triangles are congruent?

AAS, ASA, SSS, SAS

1 △

~~SSA~~

0, 1, 2

General Triangle Trigonometry: In this unit we will round all answers to the nearest tenth. Use given or exact values whenever possible. Any calculator may be used; calculator needs to be in **degree** mode.

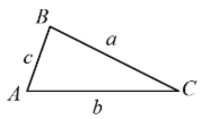
We will learn to solve oblique (non-right) triangles using the Law of sines and Law of Cosines.

The Law of Sines can be used to solve AAS, ASA, and SSA (special case) triangles.

The Law of Sines

In any triangle ABC,

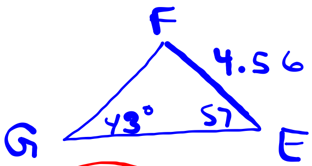
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Solve the triangles using the given information.

Example 1: In ΔEFG, $g = 4.56$, $G = 43°$, and $E = 57°$. Solve the triangle.

AAS → Exactly 1 △



$E = 57°$ $e = 5.6$

$F = 80°$ $f = 6.6$

$G = 43°$ $g = 4.56$

$\Sigma = 180°$

$\frac{e}{\sin 57°} = \frac{4.56}{\sin 43°}$

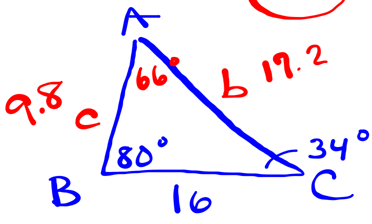
$\frac{e \sin 43°}{\sin 43°} = \frac{4.56 \sin(57°)}{\sin 43°}$

$e = 5.6$

$\frac{f}{\sin 80°} = \frac{4.56}{\sin 43°}$

$f = \frac{4.5 \sin 80°}{\sin 43°} \approx 6.6$

Example 2: In ΔABC , a = 16, B = 80°, and C = 34°. Solve the triangle.



$$\frac{b}{\sin 80^\circ} = \frac{16}{\sin 66^\circ}$$

$$b = \frac{16 \sin 80^\circ}{\sin 66^\circ}$$

$$b \approx 17.2$$

A = 66° a = 16

B = 80° b = 17.2

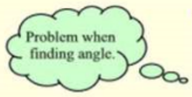
C = 34° c = 9.8

$$\frac{c}{\sin 34^\circ} = \frac{16}{\sin 66^\circ}$$

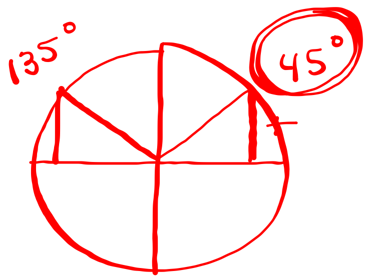
$$c = \frac{16 \sin 34^\circ}{\sin 66^\circ}$$

By definition, the word **ambiguous** means *open to two or more interpretations.*
Such is the case for certain solutions when working with the Law of Sines.

- If you are given two angles and one side (ASA or AAS), the Law of Sines will nicely provide you with ONE solution for a missing side.
- Unfortunately, the Law of Sines has a problem dealing with SSA. If you are given two sides and one angle (where you must *find an angle*), the Law of Sines could possibly provide you with one or more solutions, or even no solution.



$\sin\theta = .7071$



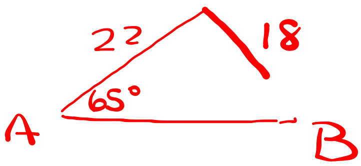
Facts we need to remember:

1. In a triangle, the sum of the interior angles is 180°.
2. No triangle can have 2 obtuse angles.
3. The sine function has a range of $[-1,1]$.
4. If $0 < \sin\theta < 1$, then θ can lie in the first quadrant (acute angle) or second quadrant (obtuse angle).

Determine if the given information supports 0 triangles, 1 unique triangle, or 2 triangles that are not congruent. If 1 or 2 triangles exist, solve the triangle(s).

Example 3: A = 65°, a = 18, and b = 22

SSA → 0 triangles



$$\frac{22}{\sin B} = \frac{18}{\sin 65^\circ}$$

$$\frac{\cancel{18}\sin B}{\cancel{18}} = \frac{22 \sin 65^\circ}{18}$$

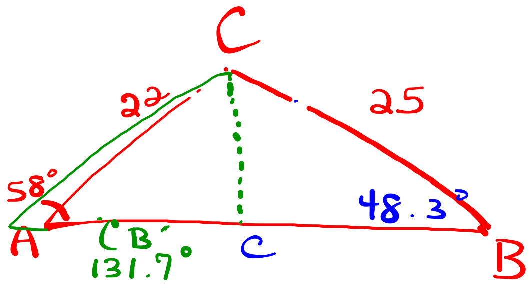
$$\sin B = \frac{22 \sin 65^\circ}{18} \approx 1.108$$

$$B = \sin^{-1}\left(\frac{22 \sin 65^\circ}{18}\right)$$

$B = \sin^{-1}(\text{ANS}) \rightarrow$ 2 Domain Error

No triangle exists.

Example 4: $A = 58°$, $a = 25$, and $b= 22$ SSA



$$\frac{22}{\sin B} = \frac{25}{\sin 58°}$$

$$\sin B = \frac{22 \sin 58°}{25}$$

$$B = \sin^{-1}(\text{ANS})$$

$$\boxed{\angle B = 48.3°;\ \angle C = 73.7° \quad c = 28.3}$$ ✓

$$\frac{c}{\sin 73.7°} = \frac{25}{\sin 58°}$$

$$c = \frac{25 \sin 73.7°}{\sin 58°}$$

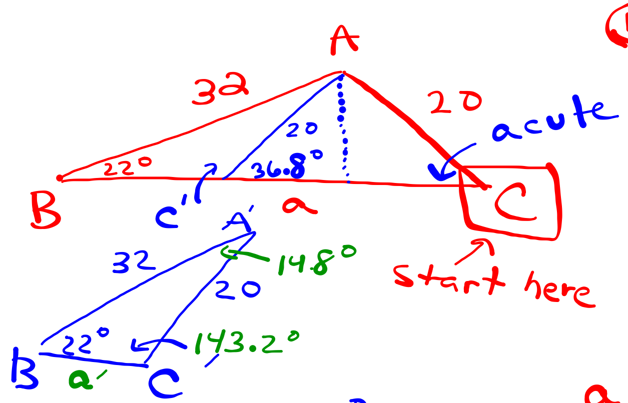
$B' = 180° - B = 131.7°$

$A + B' < 180°$

$58 + 131.7° = 189.7°$

No 2nd $\triangle$

Example 5: $B = 22°$, $b = 20$, and c= 32 *Different labels!



① $$\frac{32}{\sin C} = \frac{20}{\sin 22°}$$

$$\sin C = \frac{32 \sin 22°}{20}$$

$$C = \sin^{-1}(\text{ANS})$$

$$\boxed{\angle C \approx 36.8° \quad a = 45.7 \quad \angle A = 121.2°}$$

$$\frac{a}{\sin 121.2°} = \frac{20}{\sin 22°} \qquad a = \frac{20 \sin 121.2°}{\sin 22°}$$

$a \approx 45.7$

$C' = 180° - 36.8°$

$C' = 143.2°$

$22° + 143.2° = 165.2°$

$A' = 180° - 165.2° = 14.8°$

$$\boxed{\angle C' = 143.2,\ A' = 14.8° \quad a' = 13.6}$$

2nd $\triangle$

$$\frac{a'}{\sin 14.8°} = \frac{20}{\sin 22°}$$

$$a' = \frac{20 \sin 14.8°}{\sin 22°} \approx 13.6$$

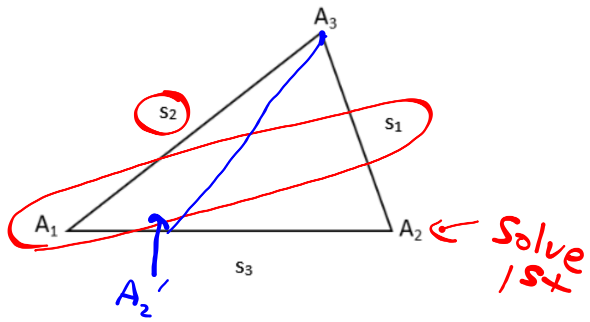
<u>Summary</u>

First, draw the triangle with given information and determine if you have AAS, ASA, or SSA.
Exactly one triangle is formed when given information for AAS or ASA triangles.
Refer to the given diagram. In the case of SSA, given angle A_1, side s_1, and side s_2, set up your triangle as below, try to draw angle A_1 near to the given information (not to scale but representative).

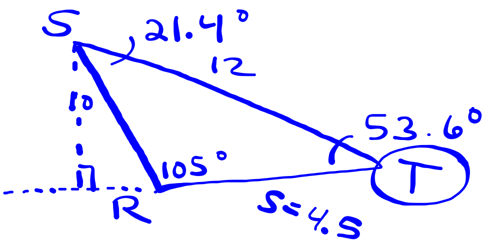
1. Find A_2 using law of sines. If $\sin A_2 > 1$, then angle A_2 does not exist and no triangle is formed. If you try to find the arcsine, the calculator will give a <u>domain</u> error.

2. If $0 < \sin A_2 < 1$, then angle A_2 is acute; <u>at least 1</u> triangle is formed. Find the measure of angle A_2 and then you can solve for angle A_3 and side s_3.

3. To determine if 2 triangles are formed, find A_2' where $A_2' = 180 - A_2$. If $A_1 + A_2' < 180$, then 2 triangles can be found. Find angle A_3' and side s_3'.



Example 6: In ΔRST, $R = 105^\circ$, $r = 12$, and t= 10. Determine the number of triangles that can be formed and solve accordingly. SSA



$$\frac{10}{\sin T} = \frac{12}{\sin 105^\circ}$$

$$\sin T = \frac{10 \sin 105^\circ}{12}$$

$$\measuredangle T = \sin^{-1}(\text{ANS}) \approx 53.6^\circ$$

$$\measuredangle S \approx 21.4^\circ$$

$$\frac{s}{\sin 21.4^\circ} = \frac{12}{\sin 105^\circ}$$

$$s = \frac{12 \sin 21.4^\circ}{\sin 105^\circ} \approx 4.5$$

$$\boxed{\begin{array}{l} \measuredangle S = 21.4^\circ \\ \measuredangle T = 53.6^\circ \\ s = 4.5 \end{array}}$$

Cannot have 2 obtuse ∡s in a Δ, so 1 Δ only.

1) 14   2) 8   3) 17   4) 29.1

5) 33   6) 53.8°   7) 43.1°   8) 13°

9) 15°   10) 59°   #11-18 Answers next page.

Using ∡B = 129°. If you used ∡C = 129°, then ∡A = 20.0.

19) 1 Δ   20) 2 Δs   21) 1 Δ

22) 1 Δ   23) 2 Δs   24) No Δ

25) 59.8 cm^2 26) 12 Km^2 27) 8.2 mi^2 28) 23 mi^2

Solve each triangle. Round your answers to the nearest tenth.

11) $m\angle A = 70°$, $c = 26$, $a = 25$

$m\angle B = 32.2°$, $m\angle C = 77.8°$, $b = 14.2$
Or $m\angle B = 7.8°$, $m\angle C = 102.2°$, $b = 3.6$

12) $m\angle B = 45°$, $a = 28$, $b = 27$

$m\angle C = 87.8°$, $m\angle A = 47.2°$, $c = 38.2$
Or $m\angle C = 2.2°$, $m\angle A = 132.8°$, $c = 1.5$

13) $m\angle C = 145°$, $b = 7$, $c = 33$

$m\angle A = 28°$, $m\angle B = 7°$, $a = 27$

14) $m\angle B = 73°$, $a = 7$, $b = 5$

Not a triangle

15) $m\angle B = 117°$, $a = 16$, $b = 38$

$m\angle C = 41°$, $m\angle A = 22°$, $c = 28$

16) $m\angle B = 84°$, $a = 18$, $b = 9$

Not a triangle

17) $m\angle B = 105°$, $b = 23$, $a = 14$

$m\angle C = 39°$, $m\angle A = 36°$, $c = 15$

18) $m\angle C = 13°$, $m\angle A = 22°$, $c = 9$

$m\angle B = 145°$, $a = 15$, $b = 22.9$