II. Solving Quadratic Equations by Completing the Square

To solve a quadratic equation in standard form $ax^2+bx+c=0$ by completing the square:

Steps

1. 'a' MUST equal 1.

2. First you move the 'c'.

3. Then you find the 'b'.
   - Then you half it.
   - Then you square it.
   - Then you add it to both sides.

4. Rewrite the perfect square trinomial as a binomial squared: $\left(x+\frac{1}{2}b\right)^2$ or $\left(x+\frac{b}{2}\right)^2$.

5. Take the square root of both sides; don't forget $\pm$.

6. Finish solving for *x* using inverse operations. Simplify radical if needed.

Example 1:

$x^2-10x+34=0$

$x^2-10x+25=-34+25$

-10

-5

25

$x^2-10x+25=-34+25$

$\sqrt{(x-5)^2}=\pm\sqrt{-9}$ IO

$x-5=\pm 3i$

$x=5\pm 3i$

$x=5+3i,\ 5-3i$

Steps

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Example 2: $\frac{2x^2+12x-10}{2}=\frac{0}{2}$

$x^2+6x-5=0$

$x^2+6x+9=5+9$

$\sqrt{(x+3)^2}=\pm\sqrt{14}$

$x+3=\pm\sqrt{14}$

$\boxed{x=-3\pm\sqrt{14}}$

Solve each quadratic equation by completing the square.

1. $x^2 - 10x + 10 = 0$

$x^2-10x+25=-10$

$(x-5)^2=\sqrt{15}$

$x-5=\pm\sqrt{15}$

$x=5\pm\sqrt{15}$

2. $x^2 + 6x + 13 = 0$

$x^2+6x+9=-13+9$

$(x+3)^2=\pm 4$

$x+3=\pm 2i$

$x=-3\pm 2i$

3. $\frac{3x^2}{3} + \frac{36x}{3} = \frac{-42}{3}$

$x^2+12x+36=-14+36$

$x^2+12x+36=22$

$(x+6)^2=\pm\sqrt{22}$

$x+6=\pm\sqrt{22}$

$x=-6\pm\sqrt{22}$

4. $\frac{5x^2}{5} - \frac{80x}{5} = \frac{-40}{5}$

$x^2-16x=-8$

$x^2-16x+64=-8+64$

$(x-8)^2=\sqrt{56}$

$x+8=\pm\sqrt{56}$

$x=-8\pm\sqrt{56}$

$x=-8\pm 2\sqrt{14}$

Assignment: Complete #8-12. Use #26-29 for additional practice (optional).

Solve each equation by completing the square.

8. $x^2 - 6x = -4$
9. $x^2 + 8 = 6x$
10. $2x^2 - 20x = 8$
11. $x^2 = 24 - 4x$
12. $10x + x^2 = 42$
13. $2x^2 + 8x - 15 = 0$

Solve each equation by completing the square.

26. $x^2 + 2x = 7$
27. $x^2 - 4x = -1$
28. $2x^2 - 8x = 22$
29. $8x = x^2 + 12$
30. $x^2 + 3x - 5 = 0$
31. $3x^2 + 6x = 1$

III. Solving Quadratic Equations using the Quadratic Formula

Given a quadratic equation in standard form, $ax^2+bx+c=0$, solutions may be found using the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ ← discriminant

1. $x^2-10x+34=0$

$a=1$ $b=-10$ $c=34$

$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(34)}}{2(1)}$

$=\frac{10\pm\sqrt{100-136}}{2}$

$=\frac{10\pm\sqrt{-36}}{2}$ $\frac{10\pm6i}{2}$

$=\frac{10}{2}\pm\frac{6}{2}i$ $x=5\pm3i$

2. $2x^2+12x-10=0$

GCF

$\frac{2}{2}$ $\frac{}{2}$ $\frac{0}{2}$

$x^2+6x-5=0$

$a=1$ $b=6$ $c=-5$

$x=\frac{-6\pm\sqrt{6^2-4(1)(-5)}}{2(1)}$

$=\frac{-6\pm\sqrt{36+20}}{2}$

$=\frac{-6\pm\sqrt{56}}{2}$ $\frac{-6\pm2\sqrt{14}}{2}$

$x=-3\pm\sqrt{14}$

3. $2x^2=x-2$

$ac=4$ $b=-1$

$2x^2-x+2=0$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(2)}}{2(2)}$

$=\frac{1\pm\sqrt{1-16}}{4}$

$=\frac{1\pm\sqrt{-15}}{4}=\frac{1\pm\sqrt{15}i}{4}$

$x=\frac{1}{4}\pm\frac{\sqrt{15}}{4}i$

4. $3x^2+5x=2$

$3x^2+5x-2=0$

$x=\frac{-5\pm\sqrt{5^2-4(3)(-2)}}{2(3)}$

$=\frac{-5\pm\sqrt{25+24}}{6}$

$=\frac{-5\pm\sqrt{49}}{6}=\frac{-5\pm7}{6}$

$x=-2,\ \frac{1}{3}$

$(3x-1)(x+2)=0$

$3x-1=0$ $x=-2$

$3x=1$

$x=\frac{1}{3}$

$\frac{-5+7}{6}=\frac{2}{6}=\frac{1}{3}$

$\frac{-5-7}{6}=\frac{-12}{6}=-2$

Discriminant of a Quadratic Equation

The discriminant of a quadratic equation in standard form, $ax^2+bx+c=0$, is the expression b^2-4ac. The discriminant of a quadratic equation tells us about the number and type of solutions of the equation.

If $b^2-4ac<0$ (negative) there are two imaginary solutions.

What is the discriminant in problem #1? -36 What are the solutions? $5\pm 3i$

If $a=1$ and b is even, solve using completing the square, otherwise use the quadratic formula.

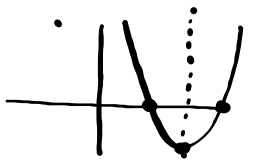
If $b^2-4ac=0$, there is one repeated, real, rational solution.

What is the discriminant of $9x^2+6x+1=0$? 0

b^2-4ac
$6^2-4(9)(1)$
$36-36$
0

Solve by factoring as a **special product.**

$(3x+1)(3x+1)=0$
$x=-\frac{1}{3}$ mult. 2



$y=ax^2+bx+c$

$x=\frac{-b\pm\sqrt{0}}{2a}$

$\frac{-b\pm 0}{2a}=\frac{-b}{2a}$

If $b^2-4ac>0$ (positive) there are two real solutions. You also need to determine if the real solutions are rational or irrational.

If b^2-4ac is a positive perfect square, the real solutions are rational, and the equation can solved by **factoring**.

What is the discriminant in problem #4? 49 What are the solutions? $x=\frac{1}{3}, -2$

If b^2-4ac is positive and **not** a perfect square, the real solutions are irrational (there will be a radical in the answer).

What is the discriminant in problem #2? 56 What are the solutions? $x=-3\pm\sqrt{14}$

If $a=1$, b is even, solve by completing the square. But if $a>1$, or b is odd, solve using the quadratic formula.

Discriminant Summary: Evaluate b^2-4ac

If the discriminant is	# of Solutions	Real or Imaginary	Rational, Irrational, or Neither	Solving Method
Negative	2	Imag.	N	CS or QF a=1, b is even All others
Zero	1 rep.	Real	Rational	SP
Positive and a perfect square	2	Real	Rational	Factoring
Positive, not a perfect square	2	Real	Irrational	CS or QF

Find the discriminant of each quadratic equation and complete the table. Do NOT solve.

Equation	Discriminant b^2-4ac	# of Solutions	Type of Solutions, list all that apply: real, imaginary, rational, irrational	Solving Method
1. $x^2+14x-2=0$	204	2	Real-Irrational	CS
2. $x^2-10x+16=0$	36	2	Real Rational	Factoring
3. $x^2+16x+64=0$	0	1 Repeating	Real Rational	Special Product
4. $2x^2-7x+1=0$	41	2	Real Irrational	CS or QF

196−4(1)(−2)
196+8=
204

(−10)²−4(1)(16)
100−64
36

16²−4(1)(64)
256−256
0

(−7)²−4(2)(1)
49−8
41