

Accelerated Precalculus

Name \_\_\_\_\_

Solving 3 X 3 Linear Systems Algebraically

Date \_\_\_\_\_ Block \_\_\_\_\_

**Recall: Solve a system of equations by elimination**

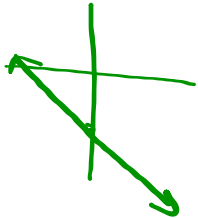
1.) Solve the following system by elimination.

$$x - 3y = -19$$

$$2x + y = -3$$

$\times 3$

$$y = -2x - 3$$



$$x - 3y = -19$$

$$6x + 3y = -9$$

$$7x = -28$$

$$x = -4$$

$$2(-4) + y = -3$$

$$-8 + y = -3$$

$$y = 5$$

$$(-4, 5)$$

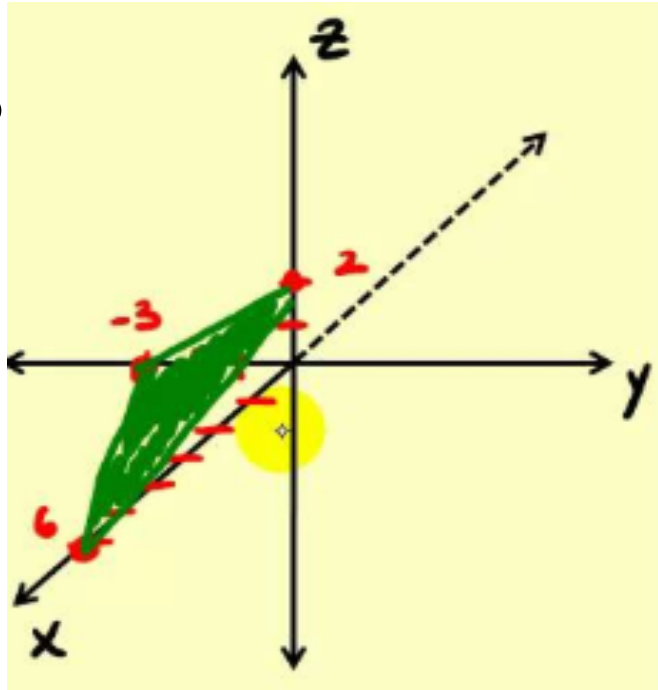
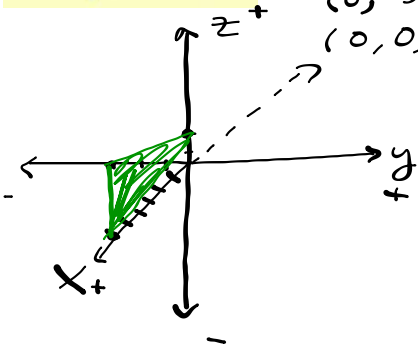
$$-4 - 3(5) = -19 \checkmark$$

$$2(-4) + 5 = -3 \checkmark$$

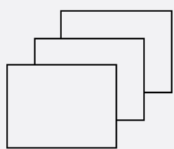
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$$x - 2y + 3z = 6$$

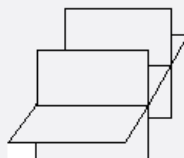
- $(6, 0, 0)$
- $(0, -3, 0)$
- $(0, 0, 2)$



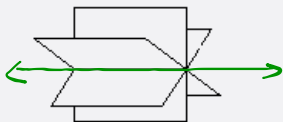
In 3D, three planes  $\pi_1$ ,  $\pi_2$  and  $\pi_3$  can intersect (or not) in the following ways:



All three planes are parallel

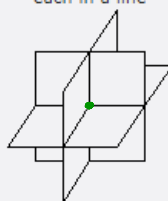


Just two planes are parallel, and the 3rd plane cuts each in a line

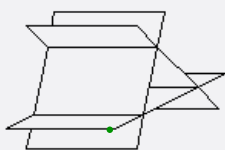


Inf. Many

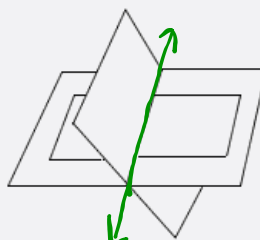
The intersection of the three planes is a line



The intersection of the three planes is a point



Each plane cuts the other two in a line



Two Coincident Planes and the Other Intersecting Them in a Line

2.) New: Solve a 3 X 3 Linear System using Elimination

- First, eliminate the same variable from 2 pairs of 2 equations (each equation should have been used at least once).
- Then solve the 2 X 2 system that remains.
- Finally, substitute the solutions of the 2 X 2 into one of the original equations to find the value of the last variable.
- Write your answer as an ordered triple,  $(x, y, z)$ . Check the ordered-triple solution by substituting values in to each equation.

$$\begin{array}{l} 2(x - y - z) = 7 \cdot 2 \quad A \\ -x + 2y - 3z = -12 \quad B \\ 3x - 2y + 7z = 30 \quad C \end{array}$$

④

$$\begin{array}{l} 7 - y - 1 = 7 \\ -y + 6 = 7 \\ -y = 1 \\ y = -1 \end{array}$$

$(7, -1, 1)$

① B+C

$$\begin{array}{r} -x + 2y - 3z = -12 \\ 3x - 2y + 7z = 30 \\ \hline 2x + 4z = 18 \end{array}$$

$$2x + 4z = 18 \rightarrow$$

③

$$\begin{array}{r} x + 2z = 9 \\ -(x - 5z) = 2(-1) \end{array}$$

② 2A+B

$$\begin{array}{r} 2x - 2y - 2z = 14 \\ -x + 2y - 3z = -12 \\ \hline x - 5z = 2 \end{array}$$

$$\begin{array}{r} x + 2z = 9 \\ -x + 5z = -2 \\ \hline 7z = 7 \\ z = 1 \end{array}$$

$$\begin{array}{r} x + 2(1) = 9 \\ x + 2 = 9 \\ x = 7 \end{array}$$

3.) Solve the system using elimination.

$$\begin{array}{r} x - y + z = 0 \quad A \\ -x + 2y - 3z = -5 \quad B \\ 2x - 3y + 5z = 8 \quad C \end{array}$$

$$\begin{array}{l} \textcircled{4} \quad x - 1 + 3 = 0 \\ \quad \quad x + 2 = 0 \\ \quad \quad x = -2 \end{array}$$

$$\boxed{(-2, 1, 3)}$$

$$\begin{array}{l} \textcircled{1} \quad \underline{A+B} \\ x - y + z = 0 \\ -x + 2y - 3z = -5 \\ \hline y - 2z = -5 \end{array}$$

$$\begin{array}{l} \textcircled{2} \quad \underline{2B+C} \\ -2x + 4y - 6z = -10 \\ 2x - 3y + 5z = 8 \\ \hline y - z = -2 \end{array}$$

$$\begin{array}{l} \textcircled{3} \quad y - 2z = -5 \\ \quad -y + z = +2 \\ \hline -z = -3 \\ \quad z = 3 \end{array}$$

$$\begin{array}{l} y - 2(3) = -5 \\ y - 6 = -5 \\ y = 1 \end{array}$$

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Recall: Solve a system of equations by substitution

4.) Solve the following system by substitution.

$$2x - 3y = -2$$

$$4x + y = 24 \rightarrow y = -4x + 24$$

$$2x - 3(-4x + 24) = -2$$

$$2x + 12x - 72 = -2$$

$$14x = 70$$

$$x = 5$$

$$2(5) - 3y = -2$$

$$10 - 3y = -2$$

$$-3y = -12$$

$$y = 4$$

$$(5, 4)$$

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5.) New: Solve a 3 X 3 Linear System using Substitution.

- First, solve for one of the variables in one of the equations.
- Substitute the expression for that variable into each of the (two) remaining equations. Simplify; you should have 2 equations with 2 variables.
- Solve the 2 X 2 system. Substitute both values back into the first equation you used for substitution at the beginning to find the last value.
- Write your answer as an ordered triple,  $(x, y, z)$ . Check the ordered-triple solution by substituting values in to each equation.

$$\begin{aligned} x - 3y - 3z &= 28 & \text{A} \\ -6x + 2y + 2z &= -24 & \text{B} \\ x &= -6y - 29 & \text{C} \end{aligned}$$

\*Complete #6 before #5!

① C → A

$$-6y - 29 - 3y - 3z = 28$$

$$-9y - 3z - 29 = 29$$

$$-9y - 3z = 57 \rightarrow -3y - z = 19$$

$$38y + 2z = -198 \rightarrow 19y + z = -99$$

③

$$\begin{aligned} 16y &= -80 \\ y &= -5 \end{aligned}$$

② C → B

$$-6(-6y - 29) + 2y + 2z = -24$$

$$36y + 174 + 2y + 2z = -24$$

$$38y + 2z = -198$$

$$-3(-5) - z = 19$$

$$15 - z = 19$$

$$-z = 4$$

$$z = -4$$

④  $x = -6y - 29$   
 $x = -6(-5) - 29$   
 $x = 1$

$(1, -5, -4)$

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Which method did you prefer? Note: The overall goal is to choose the best method for solving.

6.) Solve by substitution.

$$\begin{aligned} 5x + 2y + 2z &= 17 & \text{A} \\ -5x + 5y - z &= 30 & \text{B} \\ z &= y - 1 & \text{C} \end{aligned}$$

① C → A

$$5x + 2y + 2(y - 1) = 17$$

$$5x + 2y + 2y - 2 = 17$$

$$5x + 4y = 19$$

② C → B

$$-5x + 5y - (y - 1) = 30$$

$$-5x + 5y - y + 1 = 30$$

$$-5x + 4y = 29$$

③  $5x + 4y = 19$   
 $-5x + 4y = 29$

$$8y = 48$$

$$y = 6$$

$$5x + 4(6) = 19$$

$$5x + 24 = 19$$

$$5x = -5$$

$$x = -1$$

④  $z = 6 - 1 = 5$

$(-1, 6, 5)$

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